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Closed timelike curves and holography in compact plane waves

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Abstract: We discuss plane wave backgrounds of string theory and their relation to Gödel-like universes. This involves a twisted compactification along the direction of propagation of the wave, which induces closed timelike curves. We show, however, that no such curves are geodesic. The particle geodesics and the preferred holographic screens we find are qualitatively different from those in the Gödel-like universes. Of the two types of preferred screen, only one is suited to dimensional reduction and/or T-duality, and this provides a "holographic protection" of chronology. The other type of screen, relevant to an observer localized in all directions, is constructed both for the compact and non-compact plane waves, a result of possible independent interest. We comment on the consistency of field theory in such spaces, in which there are closed timelike (and null) curves but no closed timelike (or null) geodesics.

Keywords: D-branes, Penrose limit and pp-wave background, Space-Time Symmetries.

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1. Introduction

The four-dimensional Gödel universe [1] is a topologically trivial homogeneous space with non-zero rotation. This gives rise to some unusual properties: not only do all observers see themselves as the centre of rotation, but there exist closed timelike curves (CTCs) through every point. As an example of a space with CTCs for all times, it is unclear as to what extent Hawking's chronology protection conjecture [2], concerning the impossibility of *forming* a CTC in nature, is applicable.

For this reason, the discussion of the physics of chronology protection (see, e.g., [3] for a recent review), has mostly avoided the Gödel universe. However, the discovery [4] of supersymmetric Gödel-like solutions to five-dimensional minimal supergravity, and to its eleven-dimensional M-theoretic lift, has forced the issue, at least within the string theory community. Surprisingly, these solutions to string and M-theory turn out [5, 6] to be related to another supersymmetric space of much recent interest, namely the plane wave [7]–[9], and this is the relation of interest to us here. We should also note that supergravity solutions describing supersymmetric deformations of the extreme five-dimensional Reissner-Nordstrom black hole have been studied in this context. Both of the deformations discussed in [10] — one corresponding to the rotating black hole of [11], the other to a black hole in a Gödel-like universe — have CTCs, the precise nature of which has been examined in some detail [10] and [12]–[15]. Other non-supersymmetric solutions of interest, describing black holes in a Gödel-like universe, have recently been discussed in [16].

It turns out that these Gödel-like universes (GLUs) are related to *compactified* plane waves (CPWs) in two different ways. The eleven-dimensional CPW, dimensionally reduced on an everywhere spacelike circle, yields a ten-dimensional GLU [6]. On the other hand, a

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CPW in type II string theory is T-dual to a GLU times a transverse circle [5]. Depending on the specific plane wave we start with, these procedures give rise to a large variety of GLUs. In addition to the R-R fluxes typically supporting the CPW solutions, dimensionally reducing a CPW gives rise to a magnetic R-R one-form potential, whereas T-dualizing gives instead a NS-NS two-form potential. These background fluxes are needed to preserve supersymmetry, although the specific number of supersymmetries preserved is not invariant under the dimensional reduction or T-dualization.

In this paper we concentrate on the CPW side of the dual pair. We show in section 2 that there are CTCs in this set of geometries though, as for the GLU [5], these are not geodesic. This is a fact we find significant with respect to the consistency of propagation in this set of backgrounds. We proceed to construct, in section 3, the geodesics in the CPW background. These are needed in order to identify the holographic screens [17, 18] in the CPW geometry.

In section 4 we discuss the construction of lightsheets and of preferred holographic screens, in both compact and non-compact plane wave geometries. In both cases the lightsheets constructed are bounded by spatial surfaces, and the covariant entropy bound [17, 18] can be applied. Constructing a preferred holographic screen entails taking the union of these surfaces. In the non-compact case, this is a sensible thing to do and we exhibit the result, which may be of some interest in the study of plane wave holography [19]–[24]. In the compact case, however, we find that the region "enclosed" within the holographic screen is the complete spacetime. This is related to the existence of a compact direction, and not necessarily to the existence of CTCs. The interesting mechanism of "holographic protection" of chronology does not appear to be operating in this case — although all CTCs intersect the screen, they are also "enclosed" within it, in the precise sense defined by Bousso [17, 18].

In section 5, we comment on the relation to the similar analyses of the GLU performed in [5]. There are, in fact, two types of preferred holographic screen in our geometry. In addition to the ones discussed above, there are screens associated with the Kaluza-Klein zero modes which are "smeared" along the compact direction, just as in the GLU with flat transverse directions [5]. This smeared screen is the origin of the holographic screen in the GLU and we argue that similar statements are true in any backgrounds related by Kaluza-Klein dimensional reduction and/or T-duality.

Of course, in non-perturbative string theory, the GLUs and the CPWs are *two descriptions of a single object.* However, the approach to holography reviewed in [25] separates out the string fields into "metric" and "matter fields", as in traditional semiclassical approaches to quantum gravity. The interplay between the backgrounds described here provides hints as to how to extend the notion of holography to string backgrounds with fluxes inherited from a higher-dimensional or T-dual metric. We discuss how the covariant entropy bounds of [17, 18] can be affected by dimensional reduction and/or T-duality [1].

We conclude in section 6 with comments on attempts to quantize particles and fields in the CPW, and the related issues in the GLU. Since the only closed geodesics in these geometries are spacelike, some of the obvious problems with defining field theory in spaces

[1] For a possible relation of holography with supersymmetry, see [26, 27].

with closed timelike and/or null geodesics are avoided. In this respect, quantum field theory might be well behaved in those backgrounds. For a recent discussion of potential problems with the GLU in string theory, see [28].

While preparing this manuscript for publication, the preprint [52] appeared, which includes some results similar to those of section 3.

2. Closed timelike curves

In this section we introduce the basic spacetimes — a class of plane waves with a compact direction (CPWs), and the related Gödel-like universes (GLUs). We show that both these geometries have CTCs (as well as closed null curves). Since they are homogeneous spaces, both have CTCs through every point. We also show that none of the CTCs is a geodesic, thus avoiding some of the obvious problems such curves would generate.

2.1 Compactified plane waves

The class of metrics we are interested in can be written as

$$\mathrm{d}s_D^2 = \mathrm{d}u\mathrm{d}v - \sum_{i=1}^{N} \beta_i^2 \rho_i^2 \mathrm{d}u^2 + \sum_{i=1}^{N} \left(\mathrm{d}\rho_i^2 + \rho_i^2 \mathrm{d}\tilde{\phi}_i^2\right) + \sum_{\alpha=1}^{D-2(N+1)} \mathrm{d}x^\alpha \mathrm{d}x^\alpha \,, \tag{2.1}$$

where $\{\rho_i, \tilde{\phi}_i\}$ are polar coordinates in N transverse planes, and the light-cone coordinates are $u = z + t$, $v = z - t$. We have included as many flat directions as necessary to make this a background of string ($D = 10$) or M-theory ($D = 11$).

Note that the metric is not the pure gravitational plane wave, and must be supported by various form fields. Furthermore, for $D = 10$, this class of backgrounds gives rise to (mass)^2 terms on the string worldsheet which are all positive. String theory on such backgrounds is well understood [8, 29], as are the effects of compactification of various spacelike circles [30].

The relationship between the plane wave and the associated GLU involves dimensional reduction or T-duality along a spatial coordinate [5, 6]. This coordinate cannot be simply z, since the Killing vector

$$\xi_0 = \frac{\partial}{\partial z} = \frac{\partial}{\partial u} + \frac{\partial}{\partial v} \,, \tag{2.2}$$

is not everywhere spacelike:

$$|\xi_0|^2 = 1 - \sum_{i=1}^{N} \beta_i^2 \rho_i^2 \,. \tag{2.3}$$

Instead we can use [5] the Killing vector

$$\xi = \xi_0 - \sum_{i=1}^{N} \beta_i \frac{\partial}{\partial \tilde{\phi}_i} \qquad \Rightarrow \qquad |\xi|^2 = 1 \,, \tag{2.4}$$

which *is* everywhere spacelike.[2]

[2]Such twisted compactifications of solutions of M-theory have been much studied [31]–[33], including some which give rise to CTCs.

Identifying points a distance $2\pi nR$ along the orbits of ξ involves the following identifications on the coordinates:

$$(z, \tilde{\phi}_i) \equiv (z + 2\pi nR, \tilde{\phi}_i + 2\pi m_i - 2\pi nR\beta_i)\,. \tag{2.5}$$

Since ξ is everywhere spacelike we can compactify or T-dualize along its orbits. To facilitate this, we define the adapted coordinates

$$\phi_i = \tilde{\phi}_i + \beta_i u\,, \tag{2.6}$$

which are constant along orbits of ξ: $\xi(\phi) = 0$. The z-rotation ensures that the ϕ_i have standard periodicity, the identifications becoming

$$(z, \phi_i) \equiv (z + 2\pi nR, \phi_i + 2\pi m_i)\,. \tag{2.7}$$

In these coordinates, the metric is [5, 6]

$$\mathrm{d}s_D^2 = \mathrm{d}u\mathrm{d}v - 2\sum_{i=1}^{N} \beta_i\rho_i^2\mathrm{d}\phi_i\mathrm{d}u + \sum_{i=1}^{N}\left(\mathrm{d}\rho_i^2 + \rho_i^2\mathrm{d}\phi_i^2\right) + \sum_{\alpha=1}^{D-2(N+1)} \mathrm{d}x^\alpha\mathrm{d}x^\alpha\,, \tag{2.8}$$

where ϕ_i are angles with standard periodicity, and the Killing vector ξ is simply

$$\xi = \frac{\partial}{\partial z} = \frac{\partial}{\partial u} + \frac{\partial}{\partial v}\,. \tag{2.9}$$

Dimensional reduction or T-duality along z is now straightforward, and will be reviewed below.

We now show that there are CTCs (and closed null curves) in the above CPW, and that none of them are geodesic. Consider the curve generated by the Killing vector

$$K = \frac{\partial}{\partial z} + \sum_{i=1}^{N}\alpha_i\frac{\partial}{\partial\phi_i}\,, \tag{2.10}$$

where α_i are constant.

This curve is closed if and only if the α_i are rational multiples of $1/R$ for each i.[3] Furthermore, this curve is timelike for sufficiently large ρ_i if $\alpha_i < 2\beta_i$. This follows straightforwardly from the metric (2.8). The CTCs exist for

$$\rho_i^2 > \frac{1}{(2\beta_i - \alpha_i)\alpha_i} \geq \frac{1}{\beta_i^2}\,, \tag{2.11}$$

the latter inequality saturated for $\alpha_i = \beta_i$. By picking a rational value of α_i arbitrarily close to β_i we find CTCs arbitrarily close to $\rho_i = \beta_i^{-1}$ for any value of β_i. We therefore conclude that the CPWs considered here develop CTCs beyond the critical radii[4] $\rho_i = \beta_i^{-1}$. Note that closed null curves (CNCs) develop at the critical radii themselves, provided β_i is a rational multiple of $1/R$.

[3] To see this, put the N rational multiples over a common denominator q: $\alpha_i = p_i/(qR)$, $p_i, q \in \mathbb{Z}$. Then if we travel from z to $z+2\pi qR$ along the curve we get back to where we started: $(z, \phi_i) \equiv (z+2\pi qR, \phi_i+2\pi p_i)$.

[4] The surfaces of $\rho = \rho_{\mathrm{critical}}$ are referred to as "velocity of light surfaces" in [5].

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It is easy to see that none of these CNCs or CTCs are geodesic since, with a dot denoting differentiation with respect to an affine parameter,

$$\dot{K}^{\rho_i} + \Gamma^{\rho_i}_{\ bc} K^b K^c = \rho_i \alpha_i (2\beta_i - \alpha_i) \,, \tag{2.12}$$

all other components of the geodesic equation being trivial. Only those curves with $\alpha_i = 0$ or $\alpha_i = 2\beta_i$ (for all values of i) are geodesic and in these cases, we see from (2.11) that the CTCs are pushed off to infinity. One can, in fact, prove more generally that there are no closed timelike or null geodesics in the CPW, but we will postpone this proof until section 3, since we will need the explicit solution to the geodesic equations.

2.2 The Gödel-like universe

The CPW is related [5, 6] to a GLU by the closely related operations of T-duality or dimensional reduction along the orbits of the Killing vector ξ above. This is most easily achieved by rewriting the metric (2.8) in a form adapted to dimensional reduction along the orbits of $\partial/\partial z$:

$$\mathrm{d}s^2_D = -\left(\mathrm{d}t + \omega\right)^2 + \sum_{i=1}^{N} \left(\mathrm{d}\rho_i^2 + \rho_i^2 \mathrm{d}\phi_i^2\right) + \sum_{\alpha=1}^{D-2(N+1)} \mathrm{d}x^\alpha \mathrm{d}x^\alpha + \left(\mathrm{d}z - \omega\right)^2 \,, \tag{2.13}$$

where we define the one-form

$$\omega = \sum_i \beta_i \rho_i^2 \mathrm{d}\phi_i \,. \tag{2.14}$$

We therefore identify the dimensionally reduced metric as [6]

$$\mathrm{d}s^2_{D-1} = -\left(\mathrm{d}t + \omega\right)^2 + \sum_{i=1}^{N} \left(\mathrm{d}\rho_i^2 + \rho_i^2 \mathrm{d}\phi_i^2\right) + \sum_{\alpha=1}^{D-2(N+1)} \mathrm{d}x^\alpha \mathrm{d}x^\alpha \,, \tag{2.15}$$

describing a $(2N+1)$-dimensional GLU times $\mathbb{E}^{D-2(N+1)}$, together with an R-R 1-form potential given by $C = -\omega$.

If we think instead of the metric (2.8) as a CPW of type II string theory, one can T-dualize along orbits of $\partial/\partial z$, giving [5, 6]

$$\mathrm{d}s^2_D = -\left(\mathrm{d}t + \omega\right)^2 + \sum_{i=1}^{N} \left(\mathrm{d}\rho_i^2 + \rho_i^2 \mathrm{d}\phi_i^2\right) + \sum_{\alpha=1}^{D-2(N+1)} \mathrm{d}x^\alpha \mathrm{d}x^\alpha + \mathrm{d}z^2 \,, \tag{2.16}$$

which is just the same GLU space as above, times the T-dual circle.[5] In addition there is a NS-NS two-form potential $B = \mathrm{d}z \wedge \omega$. The theory reduced on this T-dual circle is

[5] The four-dimensional version of this metric originally appeared in [34].

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identical to the one obtained by direct reduction of the plane wave metric, since T-duality is a symmetry of the dimensionally reduced theory. The solutions which one can generate in these ways have been extensively classified in [6].

Either way, the resulting metric clearly has CTCs generated by $\partial/\partial\phi_i$ and, in both cases, some background fields are generated. These are required for unbroken supersymmetry.

We will see that the background fields change the behaviour of charged particles and fields. The charged objects are simply momentum modes in the CPW, and they become D0-branes if one dimensionally reduces an M-theory solution, or string winding modes in the T-dual GLU$\times S^1$ geometry of type II string theory.

3. Geodesics

Next we discuss the geodesics in the CPW, following closely the analysis of [5]. These include the particle geodesics discussed in the T-dual picture in [5], but also trajectories of charged particles — which would be string winding modes in the discussion of [5]. We will see that the latter behave in a way qualitatively different to that of the uncharged geodesics.

The lagrangian for a particle moving in the metric (2.8) (with a dot denoting differentiation with respect to the affine parameter λ) is

$$\mathcal{L} = \dot{u}\dot{v} - 2\sum_i \beta_i \rho_i^2 \dot{\phi}_i \dot{u} + \sum_i \left(\dot{\rho}_i^2 + \rho_i^2 \dot{\phi}_i^2\right) + P^2 = -\epsilon\,, \tag{3.1}$$

where $\epsilon = 0$ for a null geodesic, $\epsilon = 1$ for a timelike geodesic and $\epsilon = -1$ for a spacelike geodesic. We have also substituted for the $(D-2(N+1))$ conserved momenta, P_α, in the flat directions, with $P^2 = \sum_\alpha P_\alpha^2$.

There are a further $N+2$ conserved quantities coming from the Killing vectors $\partial/\partial u$, $\partial/\partial v$ and $\partial/\partial\phi_i$, the associated geodesic equations being

$$\frac{\partial\mathcal{L}}{\partial\dot{v}} = P_- \equiv \frac{1}{2}(P_z + E), \qquad \frac{\partial\mathcal{L}}{\partial\dot{u}} = P_+ \equiv \frac{1}{2}(P_z - E), \qquad \frac{\partial\mathcal{L}}{\partial\dot{\phi}_i} = 2L_i\,, \tag{3.2}$$

where, since z is compact, we will ultimately be interested in taking $P_z = n$ for some integer n. Inverting these relations gives

$$\dot{u} = P_-, \qquad \dot{v} = P_+ + 2\sum_i\left(\beta_i L_i + P_-\beta_i^2\rho_i^2\right), \qquad \dot{\phi}_i = \frac{L_i}{\rho_i^2} + \beta_i P_-\,, \tag{3.3}$$

and the resulting constraint for the radial coordinates is

$$P_-\left(P_+ + 2\sum_i \beta_i L_i\right) + \sum_i\left(\dot{\rho}_i^2 + \beta_i^2 P_-^2\rho_i^2 + \frac{L_i^2}{\rho_i^2}\right) = -(\epsilon + P^2)\,. \tag{3.4}$$

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The radial equations are

$$\ddot{\rho}_i = \frac{1}{\rho_i}\left(\frac{L_i}{\rho_i} - \beta_i P_- \rho_i\right)\left(\frac{L_i}{\rho_i} + \beta_i P_- \rho_i\right), \tag{3.5}$$

and, since the space is homogeneous, with no loss of generality we need consider only those geodesics which pass through the origin of our coordinate system. Just as in the GLU [5], it is clear from the form of the effective potential in (3.4) that this requires $L_i = 0$. We therefore obtain the following simple solutions for the radii:

$$\rho_i(\lambda) = \rho_i^{\max} \sin(\beta_i P_- \lambda), \tag{3.6}$$

where we have chosen those geodesics which pass through the origin at $\lambda = 0$. Solving (3.3) for the remaining coordinates then gives

$$\begin{aligned} t(\lambda) &= t^{(0)} + \frac{E}{2}\lambda - \frac{1}{2}\sum_i \beta_i (\rho_i^{\max})^2 \left(\beta_i P_- \lambda - \frac{1}{2}\sin(2\beta_i P_- \lambda)\right), \\ z(\lambda) &= z^{(0)} + \frac{P_z}{2}\lambda + \frac{1}{2}\sum_i \beta_i (\rho_i^{\max})^2 \left(\beta_i P_- \lambda - \frac{1}{2}\sin(2\beta_i P_- \lambda)\right), \\ \phi_i(\lambda) &= \phi_i^{(0)} + \beta_i P_- \lambda\,. \end{aligned} \tag{3.7}$$

The behaviour of the geodesics is similar to those in the GLU [5]. They spiral out from the origin, approach some maximum radius, $\rho_i^{\max}$, when $\lambda = \pi/(2\beta_i P_-)$, and re-focus back at the origin in another $\pi/(2\beta_i P_-)$ of affine distance. One can also see that t is not a good affine parameter for these geodesics. The important difference, however, is the value of the maximum radii for timelike and null geodesics. We now show that these geodesics can probe the region beyond the critical radii where the CTCs develop.

The constraint (3.4) with $L_i = 0$ can be used to put limits on $\rho_i^{\max}$. Consider for simplicity the geodesics which remain at the origin for all the planes but one,[6], that is $\rho_i^{\max} = 0$ for $i \neq 1$. Then, denoting $\beta_1 = \beta$ and $\rho_1^{\max} = \rho_{\max}$, we have

$$\rho_{\max} = \frac{1}{\beta}\sqrt{\frac{-(\epsilon + P^2 + P_+ P_-)}{P_-^2}}\,. \tag{3.8}$$

It is clear that $\rho_{\max}$ is largest (for causal propagation) when the geodesics are null, for which $\epsilon = 0$, and have vanishing momenta in the flat directions. For those geodesics one obtains

$$\rho_{\max} = \frac{\alpha}{\beta} = \frac{1}{\beta}\sqrt{\frac{E - P_z}{E + P_z}}, \tag{3.9}$$

where we have defined $\alpha = \sqrt{-P_+/P_-}$. Note that $|P_z| \le E$ gives $0 \le \alpha < \infty$.

The behaviour of the geodesics in the $\{t, \rho, \phi\}$ directions in shown in figure 1 where in this, and all other, plots we have used the dimensionless variables $\beta P_- \lambda$, βt, βz and $\beta\rho$. The Kaluza-Klein zero modes, with $P_z = 0$ ($\alpha = 1$), reach a maximum radius of β^{-1}, just as in the discussion of the GLU [5]. However, the non-zero modes, with $P_z < 0$, have

[6] In the spherically symmetric case, for which $\beta_i = \beta\ \forall i$, other geodesics are related to these by symmetry generators.

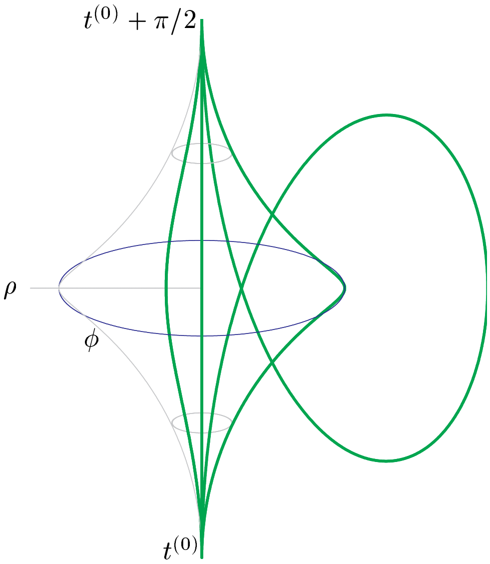

Figure 1: Null geodesics for different values of α are plotted as thick green lines. Starting at some $t = t_0$, they re-focus at $t = t^{(0)} + \pi/2$, reaching a maximal radius when $\lambda = \pi/(2\beta P_-)$. For $\alpha > 1$, geodesics can probe radii at which CTCs appear, represented by the blue circle of radius $\rho = \beta^{-1}$.

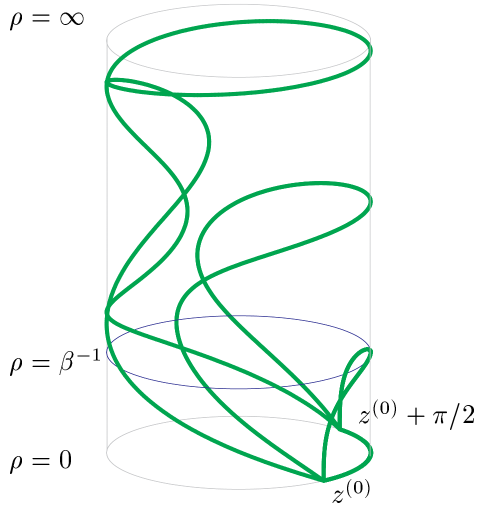

Figure 2: Geodesics for values of α are plotted as thick green lines. Starting at $\rho = 0$ and $z = z^{(0)}$, they re-focus at $\rho = 0, z = z^{(0)} + \pi/2$, reaching a maximal radius for $\lambda = \pi/(2\beta P_-)$. For $\alpha > 1$, geodesics can probe radii at which CTCs appear, represented by the blue circle at $\rho = \beta^{-1}$.

$\rho_{\max} > \beta^{-1}$. We therefore conclude that on-shell physical trajectories can probe those regions of spacetime in which CTCs appear. Indeed, for α large enough, the trajectories actually run backwards in time for some range of affine parameter.

Of course, the geodesics also move in the (compact) z direction, and we exhibit this motion in figure 2, where z runs around the circle, and ρ along the axes of the cylinder. Each point on the surface of this cylinder is a circle of radius ρ. Note that the period of βz is $2\pi\beta R$, βR being a dimensionless parameter characterizing the geometry. For the purposes of this, and all other, plots, we have (arbitrarily) chosen $\beta R = 1$.

The maximal radii for causal propagation depend on the Kaluza-Klein momentum P_z. This is represented in the reduced Gödel metric as an electric charge. The null geodesics in the CPW geometry would appear timelike in the reduced GLU geometry, representing trajectories of massive BPS particles. The interaction with the background magnetic field will be responsible for the different behaviour in that language.

Having found the general geodesic in the CPW, we can now prove that there are no closed timelike or null geodesics in this background. To see this, consider the identification of coordinates in the CPW:

$$(u, v, \phi_i) \equiv (u + 2\pi nR, v + 2\pi nR, \phi_i + 2\pi m_i)\,. \tag{3.10}$$

Due to the homogeneity of the geometry it is sufficient to consider a timelike or null geodesic starting from $(u, v, \rho, \phi_i) = (0, 0, 0, 0)$, as above. The question is then whether this geodesic can pass through the point $(2\pi nR, 2\pi nR, 0, 2\pi m_i)$. Given the equations for geodesics, (3.6) and (3.7), this would require

$$\begin{aligned} P_-\lambda &= 2\pi n R \\ P_-\beta\lambda &= 2\pi m_i \\ -\frac{(\epsilon+P^2)\lambda}{P_-} &= 2\pi n R = P_-\lambda\,. \end{aligned} \tag{3.11}$$

Note that the second equation requires β to be a rational multiple of $1/R$ — this is not required for CTCs but is necessary for closed geodesics. The third equation gives $-\epsilon = P_-^2 + P^2$, which can never be satisfied for timelike geodesics, and can only be satisfied for null geodesics when $P_- = P^2 = 0$, which does not lead to a closed geodesic. Thus, the only closed geodesics are spacelike as promised.

4. Preferred holographic screens

To construct the preferred holographic screens, we first discuss the construction of light-sheets and their bounding surfaces. As the geometry can be made only mildly curved, it is a perfect candidate for application of the ideas of [17, 18].

Since the space is homogeneous, we can concentrate on screens constructed from geodesics which emanate from the point we call our origin. The resulting screens will be observer-dependent. The holographic screen [17, 18] is defined by taking a congruence of future- or past-directed null geodesics at each point on the observer's worldline, and following them along the direction in which the expansion θ of that congruence is positive. To construct a *preferred* screen, we terminate the geodesics at the points for which θ vanishes.

We first discuss the construction for a fixed point in time along the observer's worldline. This results in a light-sheet for the observer. The covariant entropy bound [17] states that the entropy on this light sheet is bounded by the area of the orthogonal spatial surface[7] for which $\theta = 0$. The holographic screen is then simply the union of these surfaces for each point in time along the observer's worldline.

To simplify matters, we set $\beta_i = \beta\ \forall i$ and introduce an overall radial coordinate $r^2 = \sum_i \rho_i^2$ and some angles $\theta_m, m = 1, \ldots, N-1$ to replace the ρ_i. We write $\rho_i = r\hat{\rho}_i$ where the direction cosines satisfy $\sum_i \hat{\rho}_i^2 = 1$. In this coordinate system, the geodesics discussed above, namely the future-directed null geodesics which pass through the origin and which vary only in one plane, have $\theta_m =$ constant and

$$r(\lambda) = \frac{1}{\beta}\sqrt{\alpha^2 - \frac{P^2}{P_-^2}}\,\sin(\beta P_-\lambda)\,. \tag{4.1}$$

The above construction entails sending out null geodesics from the origin at each moment in time and in *all* directions. For fixed t, the surfaces at constant λ will thus be codimension two, as required. We terminate the geodesics when $\theta = 0$. Let us now drop

[7] We refer to a codimension one object as a "screen", and a codimension two object as a "surface".

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the flat directions, taking d to be the dimension of the resulting GLU space.[8] Then the surface at $\theta = 0$ is parameterized by the $d-2$ directions in which the geodesics can leave the origin. By construction, a set of coordinates for this surface is given by $\{\alpha, \theta_m, \phi_i^{(0)}\}$. Coordinates on the screen are then $\{t^{(0)}, \alpha, \theta_m, \phi_i^{(0)}\}$. One can, of course, compute θ in any coordinate system, but it is easiest to use these coordinates, which are adapted to the null congruence.

We are free to set $P_- = 1$ by rescaling λ,[9] in which case the spacetime metric is

$$\begin{aligned} \mathrm{d}s^2 &= -(1+\alpha^2)\mathrm{d}t^{(0)}\mathrm{d}\lambda - (\mathrm{d}t^{(0)})^2 - 2\frac{\alpha}{\beta}\sin(\beta\lambda) \times \\ &\quad \times \left(\cos(\beta\lambda)\mathrm{d}\alpha + \alpha\sin(\beta\lambda)\sum_{i=1}^{N}\hat{\rho}_i^2 \mathrm{d}\phi_i^{(0)}\right)\mathrm{d}t^{(0)} + \frac{\sin^2(\beta\lambda)}{\beta^2}\left(\mathrm{d}\alpha^2 + \alpha^2 \mathrm{d}\Omega_{d-3}^2\right). \end{aligned} \tag{4.2}$$

Note that the tangent

$$\xi = \frac{\partial}{\partial\lambda}, \tag{4.3}$$

to the geodesics is manifestly null, and that the coordinates on the surface $\theta = 0$, to be exhibited shortly, are manifestly orthogonal to this tangent.

The expansion of the congruence is particularly simple:

$$\theta = D \cdot \xi = (d-2)\beta\cot(\beta\lambda), \tag{4.4}$$

so that the surface for which $\theta = 0$ is thus defined by

$$\lambda = \frac{1}{\beta}\frac{\pi}{2}. \tag{4.5}$$

In terms of the original coordinates a point on the surface is given by

$$z = z^{(0)} + \frac{1}{\beta}\frac{\pi}{4}, \qquad r = \frac{\alpha}{\beta}, \qquad \phi_i = \phi_i^{(0)} + \frac{\pi}{2}, \qquad \theta_m = \text{constant}. \tag{4.6}$$

The induced metric on the holographic screen is given by substituting the condition (4.5) on λ into the metric (4.2), giving

$$\mathrm{d}s^2\big|_{\text{screen}} = -(\mathrm{d}t^{(0)})^2 - 2\alpha^2\beta\sum_{i=1}^{N}\hat{\rho}_i^2\mathrm{d}\phi_i^{(0)}\mathrm{d}t^{(0)} + \mathrm{d}\alpha^2 + \alpha^2\mathrm{d}\Omega_{d-3}^2, \tag{4.7}$$

where we have rescaled α by β. The screen has lorentzian signature, with each spacelike slice at constant $t^{(0)}$ being flat $\mathbb{E}^{d-2}$, α acting as the radial coordinate in this space. It seems to exhibit rotation, but defining the coordinates

$$\hat{\phi}_i^{(0)} = \phi_i^{(0)} - \beta t^{(0)}, \tag{4.8}$$

[8] We can easily construct a screen relevant to an observer delocalized in the flat directions, by taking $P^2 = 0$ and then smearing over these directions. (What we mean by this process of smearing will be discussed in the following section.) Accounting for non-zero P^2 complicates matters considerably, as in the analysis of [5] for the GLU.

[9] One can show that the location of the surface $\theta = 0$ is independent of the choice of λ under such rescalings.

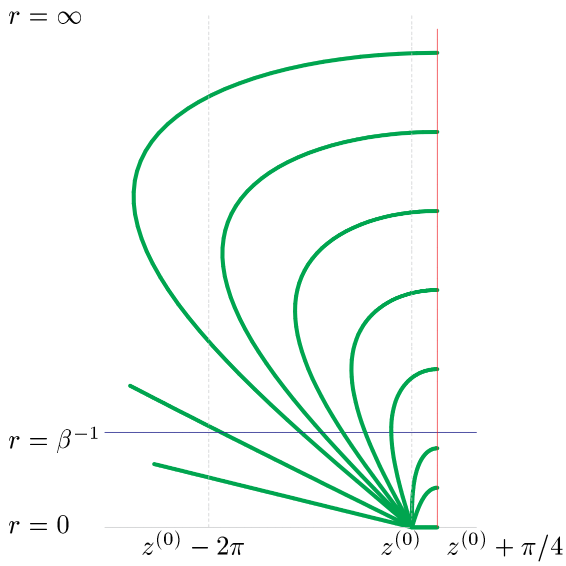

Figure 3: The holographic screen is shown in red, α being the coordinate along this line of constant $z = z^{(0)} + \pi/4$, and each point being a sphere of radius r. Geodesics for different values of α are again plotted as thick green lines, starting from the origin at $z = z^{(0)}$. For constant r, the screen is just a point in the z direction. To discuss the compact plane wave (with $\beta = 1/R$), we simply identify $z = z^{(0)}$ with the dotted lines $z = z^{(0)} \pm 2n\pi$. In this case, the blue line represents the projection of a CTC into the $\{r, z\}$ plane.

gives

$$\mathrm{d}s^2\big|_{\mathrm{screen}} = -(1 + \alpha^2\beta^2)(\mathrm{d}t^{(0)})^2 + \mathrm{d}\alpha^2 + \alpha^2 \mathrm{d}\Omega^2_{d-3}\,, \tag{4.9}$$

where now the $(d-3)$-sphere is parameterized by $\{\hat{\phi}^{(0)}_i, \theta_m\}$. For small α, the induced metric is that of flat Minkowski space. Note that the area of the surface bounding the lightsheet, at constant $t^{(0)}$, is infinite; it is just flat space. The covariant entropy bound [17, 18] thus implies that the entropy enclosed on the lightsheet can be infinite, as in AdS and Minkowski space, but unlike deSitter space.

To picture the holographic screen in spacetime, it is useful to first consider the case of the non-compact plane wave (the compact case can then be analysed by identifying points the z direction). Indeed, given the difficulties in understanding holography in the plane wave [19]–[24], this is a useful exercise in itself. All the previous analysis can be applied directly to the non-compact plane wave. In particular, the expressions (3.6), (3.7) and (4.4) for the null geodesic congruence and its expansion remain valid. Of course, there are no issues concerning chronology protection in this case since the non-compact plane wave does not contain CTCs.

Figure 3 shows the preferred holographic screen in the non-compact geometry. Since the screen extends over the entire range of the angular coordinates $\{\phi_i, \theta_m\}$ and time t, we are only concerned with its position in the r and z directions. It is represented by the red line, parameterized by the coordinate α. The thick green lines are the null geodesics which terminate at the screen, at a value of λ given by (4.5). At fixed $t^{(0)}$, these geodesics form one of the lightsheets for the screen, and it is easy to see that taking the collection of these lightsheets for all $t^{(0)}$ fills the entire region to the left of the screen; it is just the region $z < \pi/4$. We could alternatively consider the lightsheets which terminate at the caustic $z = z^{(0)} + \pi/2$. These would cover the region $z > \pi/4$.

When discussing the non-compact plane wave, one might prefer to construct the holographic screen in light-cone coordinates, using $u^{(0)} = \left(z^{(0)} + t^{(0)}\right)/2$ as the time coordinate on the screen. In these coordinates, the condition (4.5) for vanishing θ still holds. The induced metric on the screen is

$$\mathrm{d}s^2\big|_{\text{screen}} = -\alpha^2\beta^2(\mathrm{d}u^{(0)})^2 + \mathrm{d}\alpha^2 + \alpha^2\mathrm{d}\Omega^2_{d-3}\,, \tag{4.10}$$

and a point on the surface of the screen is given by

$$v = v^{(0)}, \qquad r = \frac{\alpha}{\beta}\,, \qquad \phi_i = \phi_i^{(0)} + \frac{\pi}{2}, \qquad \theta_m = \text{constant}\,. \tag{4.11}$$

If we replace z with v in figure 3, and move the screen to $v^{(0)}$, then a similar picture would still be valid.

It would be interesting to find a relation between this preferred screen and the boundary of the plane wave, which was found to be a null line in [22, 23].

The case of the *compact* plane wave is more complex. To discuss this (for the case $\beta = 1/R$), we simply identify the lines $z = z^{(0)} + 2\pi n$. Then circles on the cylinder at constant $r > \beta^{-1}$ will be the projection into the $\{r, z\}$ plane of one of the CTCs discussed in section 2.

In the GLU, as in other interesting geometries, the surface $\theta = 0$ encloses a region of space, so that the spacelike projection of [17, 18] can be applied. In this sense, the concept of holographic protection of chronology is quite appealing: spatial sections of the screen (surfaces) should encode all information, or degrees of freedom, within a volume of space. An holographic theory on the screen would describe physics in a region of spacetime which does not contain CTCs.

In our case, it is easy to see that any point in spacetime is "enclosed" within the holographic screen. The collection of lightsheets shown in figure 3 for the CPW covers the whole of spacetime. Thus, the region "cut out" by the screen is not free of causal ambiguity. Holographic protection of chronology in this case does not appear to be in operation.

5. Relation to the Gödel-like universe

When the geometry contains a Killing vector along a possibly compact direction, as in the GLU with flat transverse directions [5], there are at least two types of lightsheet and screen one can discuss. The first is that which we have constructed above, and is relevant to an observer localized in all directions. It is not translationally invariant along the compact direction, so it is difficult to see how it is connected to the lower-dimensional screen in the GLU.

However, as in [5], one may also contemplate "smearing" in the compact directions. These latter screens are constructed by taking $\alpha = 1$ or $P_z = 0$, i.e., for the Kaluza-Klein

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zero modes. Of course, α is then no longer any use as a coordinate along the screen, but since

$$\left(\frac{\partial}{\partial z^{(0)}},\xi\right)=\left(\frac{\partial}{\partial z},\xi\right)=\frac{1}{2}(1-\alpha^2)\,, \tag{5.1}$$

the vector $\partial/\partial z_0$ is orthogonal to the tangent ξ when $\alpha = 1$. For these light rays, z_0 replaces α as a coordinate on the screen. Working with these coordinates, one finds that the expansion is given by

$$\theta=\frac{\left((d-3)-(d-2)\beta^2r^2\right)}{r\sqrt{1-\beta^2r^2}}\,, \tag{5.2}$$

so that

$$r=\sqrt{\frac{d-3}{d-2}}\,\frac{1}{\beta}<\frac{1}{\beta}\qquad\Leftrightarrow\qquad\sin(\beta\lambda)=\sqrt{\frac{d-3}{d-2}}\,, \tag{5.3}$$

is the surface $\theta = 0$. The smeared screen thus provides a manifest holographic protection of chronology. Note also that, unlike the unsmeared case, all null geodesics intersect the smeared screen precisely twice in each cycle (as λ runs from 0 to $\pi/(2\beta P_-)$).

It is only this smeared screen which is relevant to the GLU. It is translationally invariant along z, so can be dimensionally reduced in the usual manner. Indeed, it is easy to see that the induced metric on the smeared screen is

$$\begin{aligned}\mathrm{d}s^2\big|_{\text{screen}} &= -\left(\mathrm{d}t^{(0)}+\frac{1}{\beta}\frac{d-3}{d-2}\sum_{i=1}^{N}\hat\rho_i^2\mathrm{d}\phi_i^{(0)}\right)^2+\frac{1}{\beta^2}\frac{d-3}{d-2}\mathrm{d}\Omega_{d-3}^2+\\ &\quad+\left(\mathrm{d}z^{(0)}-\frac{1}{\beta}\frac{d-3}{d-2}\sum_{i=1}^{N}\hat\rho_i^2\mathrm{d}\phi_i^{(0)}\right)^2\,,\end{aligned} \tag{5.4}$$

which one can dimensionally reduce along $z^{(0)}$ to give the induced metric on the screen in the GLU [5], or T-dualize to give the induced metric on the smeared screen in the GLU times a circle.

A similar structure exists for a more general Kaluza-Klein dimensional reduction. Denoting the $(D-1)$-dimensional coordinates by x^a, the z equation of motion for a particle moving in the D-dimensional metric

$$\mathrm{d}s_D^2=e^{2\alpha\phi}\mathrm{d}s_{D-1}^2+e^{2\beta\phi}(\mathrm{d}z+A)^2\,, \tag{5.5}$$

is $\dot z = e^{-2\beta\phi}P_z - A_a\dot x^a$. The vector tangent to a null geodesic (in the higher dimensional sense) will have the form

$$\xi=\dot x^a\frac{\partial}{\partial x^a}+\dot z\frac{\partial}{\partial z}\,, \tag{5.6}$$

so that

$$\left(\frac{\partial}{\partial z^{(0)}},\xi\right)=\left(\frac{\partial}{\partial z},\xi\right)=P_z\,. \tag{5.7}$$

The localized screen, the position of which varies in the z direction, is parameterized by P_z. There also exists a smeared screen, however, which is parameterized by $z^{(0)}$, and for which $P_z = 0$.

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The induced metric on the smeared screen is a generalization of (5.4):

$$\mathrm{d}s^2_D\big|_{\mathrm{screen_D}} = e^{2\alpha\phi}\,\mathrm{d}s^2_{D-1}\big|_{screen_{D-1}} + e^{2\beta\phi}(\mathrm{d}z^{(0)} + A)^2\,, \tag{5.8}$$

where the fields ϕ and A should be evaluated on the $(D-1)$-dimensional screen. We see that the areas of the screens in the corresponding Planck units behaves simply under dimensional reduction, if the higher dimensional screen is smeared. Similar comments apply to geometries related by T-duality in which, at least for the case at hand, one can think of the Kaluza-Klein vector as the z components of the B-field. We note that the lower dimensional Planck scale is determined at the screen's location, in case the dilaton varies.

The relation between the localized screens in D and $D-1$ dimensions (or equivalently between the localized screen and its smeared version) is unclear. In particular, the entropy bound derived from the lower dimensional surfaces (i.e. sections of the corresponding screen) may be stronger or weaker than the covariant entropy bound of the full geometry. One may be tempted to regard the lower dimensional version as more coarse grained and the associated entropy as excluding the massive Kaluza-Klein modes; it would be useful to make this more precise.

6. Conclusions

Perhaps the most interesting question concerning the spaces which we have been discussing is whether string theory can, in some sense, "cope" with CTCs. Certainly, the very existence of highly supersymmetric string backgrounds with CTCs suggests that such geometries should not *a priori* be excluded from acceptable string vacua without a concrete physical reason. In the present context, due to special features of these backgrounds, it is still unclear as to whether quantum field theory, or string theory, can be consistently defined.[10]

Neither the GLU nor the CPW are globally hyperbolic, the classical Cauchy problem being ill-posed in both spaces.[11] As in AdS spaces, one possible approach to define dynamics on such a spacetime is to supplement the initial conditions with some boundary data. The crucial issue is whether there exists a choice of such boundary conditions which renders the resulting propagation consistent.

Unlike the AdS case, here the surface on which to define boundary data is most naturally the Cauchy horizon. Such a prescription in other instances leads to instabilities, e.g., for the Cauchy horizon in the interior of generic black holes [41]–[43]. Here no such

[10] For various efforts in the four-dimensional context, see [35, 36].

[11] In the GLU, this is intimately related to the existence of CTCs, but the lack of a Cauchy surface in the case of the CPW runs deeper: even the uncompactified plane wave, with no CTCs, is not globally hyperbolic [37]. Defining quantum field theory on this space is thus a somewhat subtle issue. The surface $u =$ constant can be used as a substitute Cauchy surface as in, e.g., [38]. This fails to capture only those geodesics which travel parallel to the wave [39]. Alternatively, as we are arguing here, one can impose a cutoff on the transverse coordinates, leaving a patch of spacetime for which the classical Cauchy problem is well-defined [40].

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instabilities manifest themselves. This is related primarily to two important facts: the spectrum of matter fluctuations is discrete (a fact inherited from the plane wave), and the CTCs are not geodesics.

One can, for example, expect the energy-momentum tensor of matter fields to diverge at the Cauchy horizon. This is not expected to be the case here, as the spectrum of modes is discrete. Related to that, a possible classical super-radiance phenomena would be a signal of instability. This requires separation of modes into incoming and outgoing, which is unnatural in a plane wave geometry.

In case of geodesic CTCs, such as in flat space with compact time, one can easily construct problematic amplitudes using the fact that CTCs dominate some quantum amplitudes in a semi-classical approximation. In our case, it is not clear that the presence of off-shell trajectories which are CTCs generates any problem for quantum mechanics in this space.

Recently, practical methods have been developed to deal with the CTCs present in certain orbifolds [44]. Although interesting, it is not clear as to what extent such methods would be applicable here: much use is made of the covering space of the orbifold, a notion which does not generalize to the case at hand.

Alternatively [45], at least in the case of the GLU [46, 47], one can analytically continue to the euclidean regime and use the techniques of euclidean field theory. Results thereby obtained can be analytically continued back to the lorentzian regime to obtain information about quantum fields in the original space. For additional discussion of QFT in non-globally hyperbolic spaces see, e.g., [48]–[50].

Using holography in these backgrounds is appealing, especially due to the need to carve out a finite region of spacetime in order to define the dynamics uniquely. However, at least in one example, we have found that the concept of "holographic protection" of chronology appears to be inadequate. It would be interesting to examine further, and in more generality, the connections between the concepts of holography and chronology protection. It would also be of interest to examine more quantatively how the processes of dimensional reduction and/or T-duality affect the covariant entropy bounds of Bousso [17, 18].

One may be tempted to use relations to uncompactified plane waves, or to AdS [51], to find the holographic theory living on the screens directly. Such attempts are not straightforward, as is demonstrated by our lack of understanding of plane wave holography [19]–[24].

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